Aspects of Coding for Molecular Communications

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Aims

- Develop tractable approximations
  - Use in larger programs
- Investigate prospects for coding
  - Given the limited resources
- Consider relaying and network coding
  - Given the molecular channel

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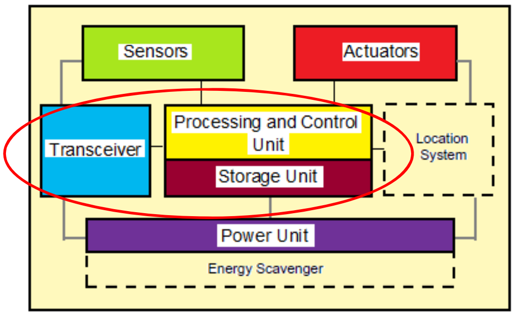
Nano-machines

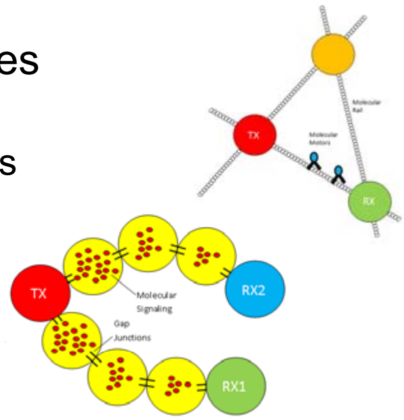
- Likely elements (Akyildiz et al. 2008)



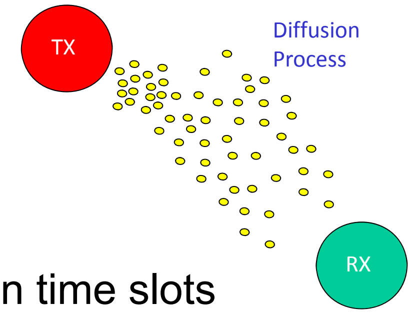
- Power?

Molecular Communications

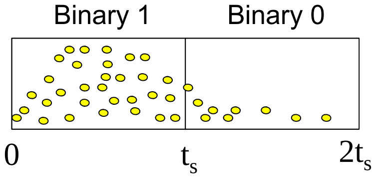
- "Wet" Techniques
  - Molecular Motors
  - Ion Gaps
  - **Diffusion**



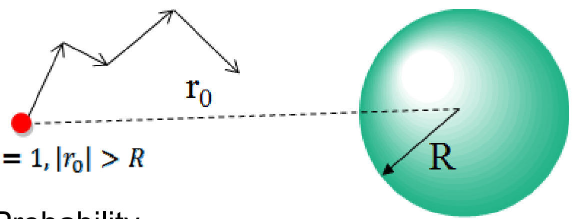
Communication using diffusion



- Bits in time slots



Modeling diffusion in 3D



$S(r_0, 0) = 1, |r_0| > R$

Survival Probability

$S(r_0, t)$

$S(|r_0| = R, t) = 0$

$S(|r_0| \to \infty, t) = 1$

Must also tie up with known probability of escape for Brownian motion

Ziff et al., 2009

Modeling diffusion in 3D

- Survival probability to time *t* satisfies backward diffusion equation

$$\frac{\partial S(r_0,t)}{\partial t} = D\nabla^2 S(r_0,t)$$

- Using radial symmetry and considering the *capture probability*

$$p_0 = \frac{R}{r_o}\operatorname{erfc}\left\{\frac{(r_0 - R)}{2\sqrt{Dt}}\right\}$$

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Hit time distribution

- Straight application of the rules of differentiation delivers

$$h(t) = \frac{R}{r_0}\frac{d}{2\sqrt{\pi D}}\frac{1}{t^{3/2}}\exp\left\{-\frac{d^2}{4Dt}\right\}$$

$$d = r_0 - R$$

This looks familiar as it should

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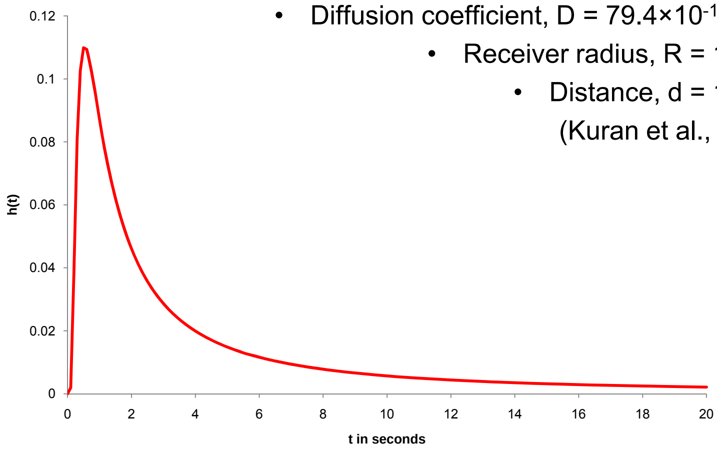
Hit time distribution plot



- Diffusion coefficient, D = 79.4×10^{-12} ms^{-1}
- Receiver radius, R = 10 μm
- Distance, d = 16 μm

(Kuran et al., 2010)

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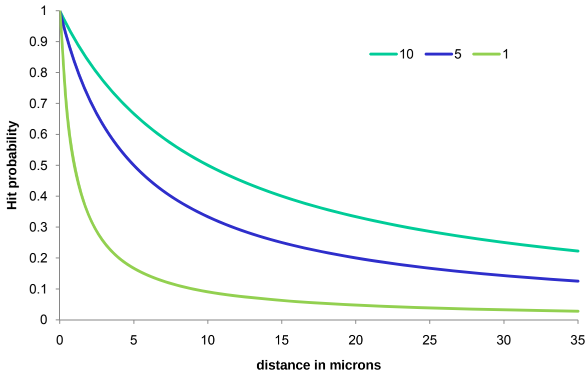
Hit probability as t →∞

- For different receiver diameters in μm



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Relationship for t_s (60% cut off)

$$\mathrm{erfc}\left\{\frac{d}{2\sqrt{Dt_s}}\right\} = 0.6$$

$$1 - \mathrm{erf}\left\{\frac{d}{2\sqrt{Dt_s}}\right\} = 0.6$$

$$t_s = \frac{d^2}{4D(erf^{-1}[0.4])^2}$$

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Comparison with Kuran et al.

- Values for 60% of molecules to arrive

Distance (μm)	t_s (s) Kuran et al.	t_s (s) in this work
1	0.03	0.023
2	0.11	0.092
4	0.4	0.366
8	1.54	1.466
16	5.9	5.862
3)	22.01	23.449

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Channel Model

- As per Kuran et al., Binomial as either the molecule arrives or it does not with hit probability $P_{hit}(d,t_s)$
- There are N molecules per bit $B\{N, P_{hit}(d,t_s)\}$
- Molecules from previous '1' bit

$$B\{N, P_{hit}(d,2t_s)\} - B\{N, P_{hit}(d,t_s)\}$$

- Gaussian but other approximations under development

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Channel Model

Using one previous bit the options are clearly:

Bit n-1	Bit n	State	Error
0	0	a	$P_{b\|a}$
0	1	b	$P_{a\|b}$
1	0	c	$P_{d\|c}$
1	1	d	$P_{c\|d}$

$$P_e = P_a P_{b|a} + P_b P_{a|b} + P_c P_{d|c} + P_d P_{c|d}$$

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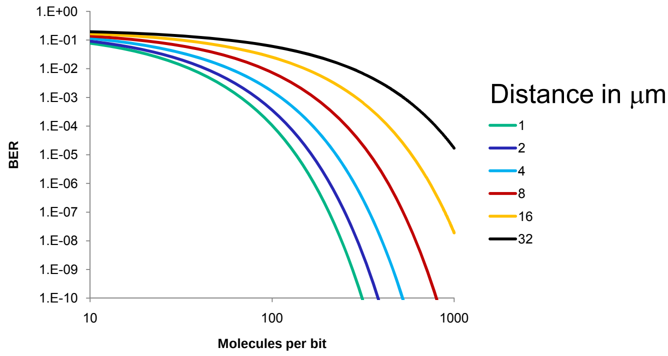
BER vs. number of molecules



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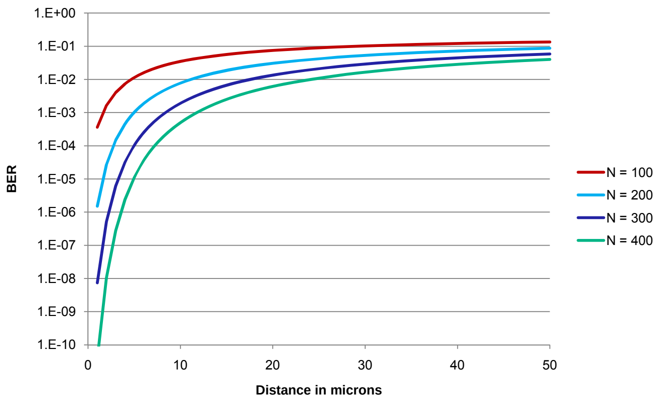
BER vs. distance



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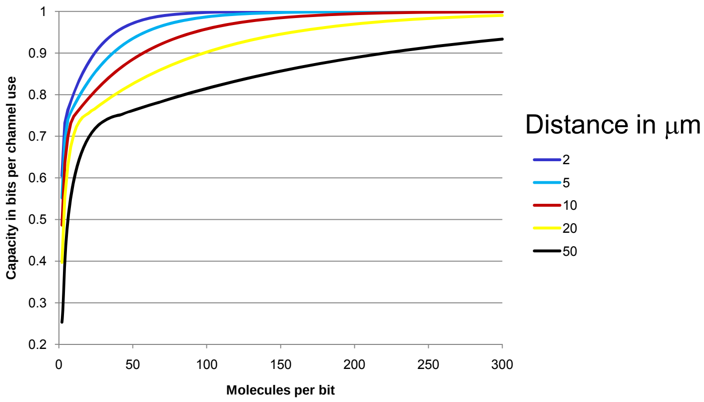
Shannon Capacity

- For decision level α: $C = \max_{0 \le \alpha \le N} \{H(Y) - H(Y|X)\}$



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The "simplest of codes"

- Repetition code 0 →000 and 1 → 111
- When just one probability of bit error

$$P_e = p^3 + 3p^2(1-p)$$

- Here each case is different
- For example 111 → 010

$$p_e = \frac{1}{2}(P_{c|d} + P_{a|b})(1 - P_{a|b})P_{c|d}$$

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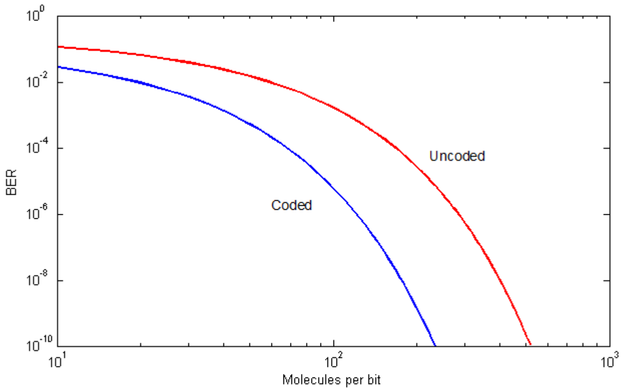
Repetition Code



Distance = 16 µm; coding gain ~2.25 dB

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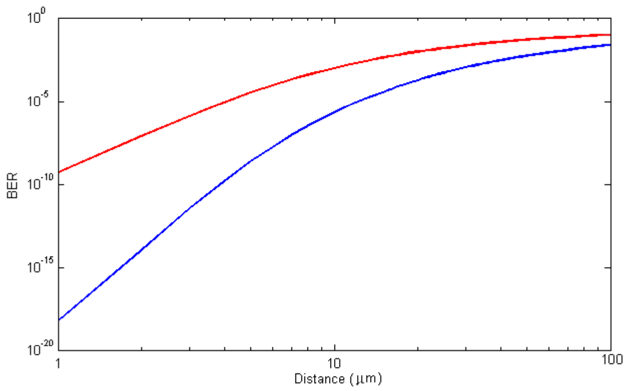
Repetition Code



N = 350; @ 10^{-9}, ~4 µm extra distance, i.e. 400% increase

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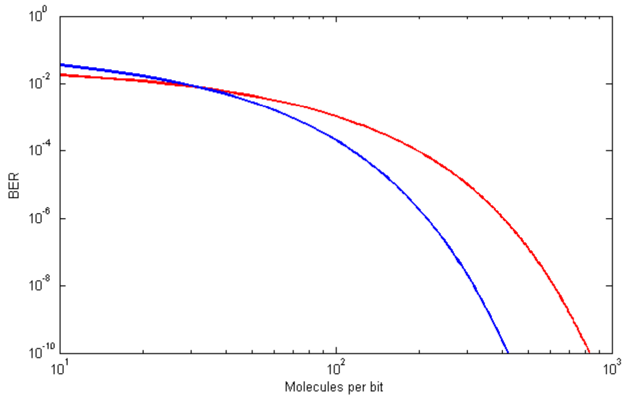
Repetition Code



With 95% ‘0’ bits classic coding crossover is seen; d = 4 μm

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Hamming Code

Simple (7,4) version corrects one error has typical generator:

$$\mathbf{G}=\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix} \qquad \mathbf{T}=\begin{bmatrix}1&0&1&0\end{bmatrix}\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix}=\begin{bmatrix}1&0&1&0&0&1&0\end{bmatrix}$$

Need weight enumerator $A(w)=1+7w^3+7w^4+w^7$

How many code words

Hamming weight

$$P_e \le 7p(3)+7p(4)+p(7)$$

$$p(w)=\begin{cases}\sum\limits_{i=\frac{w+1}{2}}^{n}\binom{n}{i}p^i(1-p)^{n-i} & w \text{ odd}\\ \frac{1}{2}\binom{n}{\frac{w}{2}}p^{\frac{w}{2}}(1-p)^{n-\frac{w}{2}}+\sum\limits_{\frac{w}{2}+1}^{n}\binom{n}{i}p^i(1-p)^{n-i} & w \text{ even}\end{cases}$$

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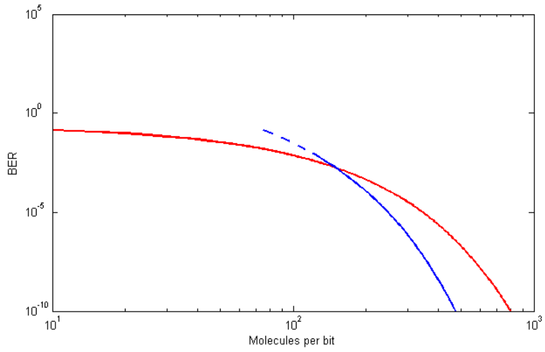
Hamming Code

Take upper bound with p = maximum value of bit error probability



Distance = 8 μm; coding gain ~2.5 dB

BUT the rate is 4/7 - better than the 1/3 for the repetition code

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Ongoing Work

- Developing Saddlepoint approximations and Modified Chernoff Bounds
- Checking the full ISI impact
- Modulation options
- Improving relatively crude bounds
- Including energy impact

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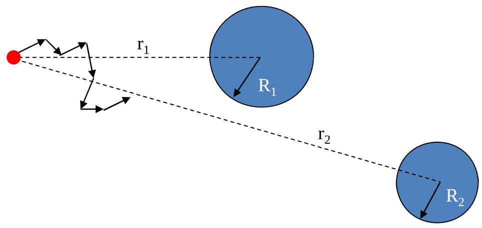
Further Scenarios

- Relaying and Network Coding
- Initially need to solve:



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Summary

- Initial modelling results avoiding lengthy simulations
- Simple codes introduced
- Useful coding gains for nanomachines
- Further development needed to move to Relaying and Network Coding

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